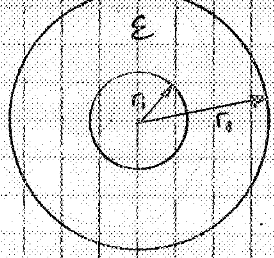
Circuit Cavity QED

Transmission line

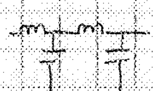


$$\vec{E}(\rho, \varphi, z) = \frac{V_0}{\ln(r_0/r_i)} \frac{\vec{e}_\rho}{\rho} e^{i\omega t - i\beta z}$$

$$\vec{H}(\rho, \varphi, z) = \frac{I_0}{2\pi} \frac{\vec{e}_\varphi}{\rho} e^{i\omega t - i\beta z} \quad \text{TEM modes}$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

$$Z_0 = \frac{V_0}{I_0} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\ln(r_0/r_i)}{2\pi} \qquad \beta = \frac{\omega}{\frac{c}{\sqrt{\varepsilon\mu}}} \quad \text{no dispersion}$$



capacitance per unit length

$$C^* = \frac{1}{V_0^2} \int_\Sigma \varepsilon_r \varepsilon_0 \vec{E}\,\vec{E}^*\,dx\,dy = \frac{2\pi}{\ln(r_0/r_i)} \varepsilon_r \varepsilon_0$$

inductance per unit length

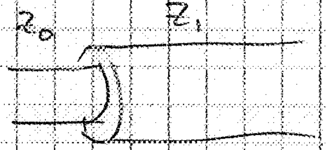
$$L^* = \frac{1}{I_0^2} \int_\Sigma \mu_r \mu_0 \vec{H}\,\vec{H}^*\,dx\,dy = \frac{\ln(r_0/r_i)}{2\pi} \mu_r \mu_0$$

$$Z_0 = \sqrt{\frac{L^*}{C^*}}$$

semi infinite transmission line is physically identical to a resistor of $R = Z_0$

LII,2

impedance missmatch

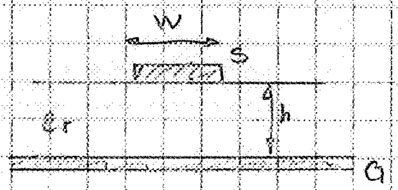


reflection coefficient

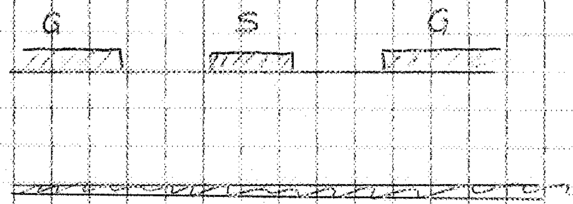
$$\Gamma = \frac{Z_1 - Z_0}{Z_1 - Z_0}$$

other transmission lines

microstrip



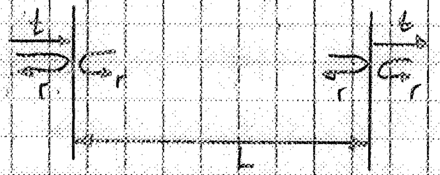
coplanar waveguide



transmission line resonator

slides

general resonance

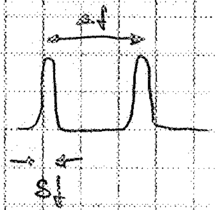


$$S = t\, e^{i\beta L}\, t$$
$$+\ t\, e^{i\beta L}\, r\, e^{i\beta L}\, r\, e^{i\beta L}\, t$$
$$+ \ldots$$

$$= t^2 e^{i\beta L} \left(\sum_n \left(r\, e^{i\beta L} \right)^{2n} \right)$$

geometrical series

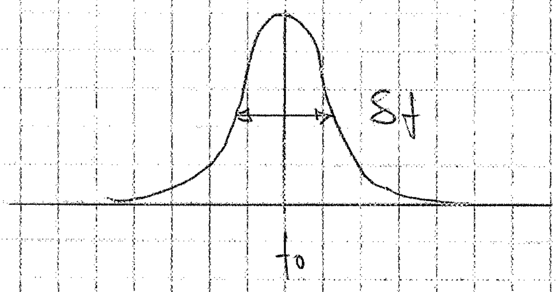
$$= a \frac{1}{1 - \left(r e^{i\beta L} \right)^2}$$



resonance if $\beta L \approx m\pi$

$$S_L = \frac{1}{f - f_0 - i\gamma}$$

$$|S_L| = \frac{1}{(f-f_0)^2 + \gamma^2}$$ Lorentz curve



δf full width halfmax

$\delta f = 2\gamma$ Quality factor $Q = \frac{f_0}{\delta f}$

Finesse $= \frac{\Delta f}{\delta f}$

L11.4

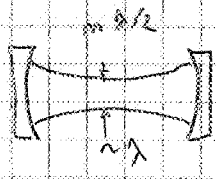
transmission line cavity

$\boxed{\text{slide}}$

$$Q_{cpl} = \frac{\pi}{4m} \frac{1}{\left(C_{cpl}\, 2\pi f_0 Z_0\right)^2} \qquad f_0 = \frac{c/\sqrt{\varepsilon_r \mu_r}}{2L}$$

$$Q_{tot}^{-1} = Q_{cpl}^{-1} + Q_{int}^{-1}$$

optical cavity



fields for one photon

$$\frac{1}{2} h f_0 = \frac{\varepsilon_0}{2} \int \vec{E}^2 \, dV \approx \frac{\varepsilon_0}{2} E_0 V$$

$$E_0 = \sqrt{\frac{h f_0}{V \varepsilon_0}}$$

cavity volume $V = \pi r^2 \lambda/2$ $\qquad \lambda = \frac{c}{f_0}$

$$\Longrightarrow \quad E_0 = \frac{f_0}{r} \sqrt{\frac{h}{\frac{\pi}{2} \varepsilon_0 c}}$$

$f_0 = 5$ GHz $\qquad r = 10\,\mu m$

$h f_0 = 3.3 \cdot 10^{-24}$ J

$E_0 = 0.2$ V/m

L11,5

comparing to conventional cavity

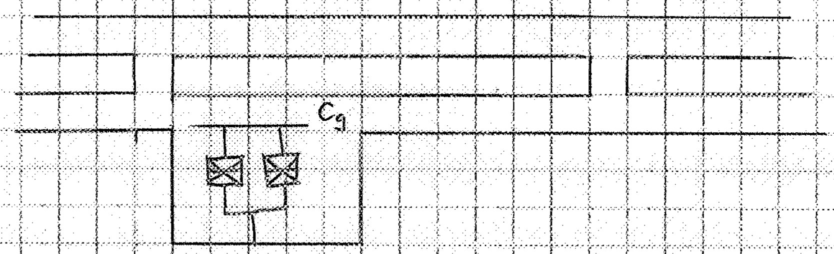
$V \propto \lambda^2$ $\quad$ E_0 is larger by factor $\frac{\lambda}{r}$

superconductivity $Q \sim 5 \quad 10^5$

$\boxed{\text{slide}}$

$$5\ \mathrm{GHz} \cdot h = k_B \cdot 240\,\mathrm{mK}$$

circuit CQED



Cooper-pair box Hamiltonian

$$H_q = -\frac{1}{2} E_{el}\,\sigma_z - \frac{1}{2} E_J\,\sigma_x$$

with $E_{el} = 4E_c\,(1 - 2n_g)$ $\quad n_g = \frac{C_g V_g}{2e}$

$$V_g = \frac{C_g}{C_\Sigma}\left(V_g^{DC} + V^{AC}\right)$$

$$V^{AC} = V^0\,(a + a^\dagger) \qquad V^0 = E^0 r$$

$$H_q = -2E_c\,(1 - 2n_g^{DC})\,\sigma_z - \frac{E_J}{2}\,\sigma_x$$

$$- e\frac{C_g}{C_\Sigma} V_0\,(a + a^\dagger)\,(1 - 2n_g^{DC} - \sigma_z)$$

In the qubit eigenbasis

$$H_{tot} = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma_z$$

$$- e\frac{C_g}{C_\Sigma} V_0\,(a + a^\dagger)\,\left(1 - 2n_g^{DC} - \cos(\theta)\,\sigma_z + \sin(\theta)\,\sigma_x\right)$$

$$\Omega = \sqrt{E_J^2 + E_{el}^2} \qquad \theta = \arctan\left(\frac{E_J}{E_{el}}\right)$$

for $n_g = \frac{1}{2}$ and neglecting fast rotating terms

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma_z + \hbar g \left(a^\dagger \sigma_- + a\sigma_+\right)$$

Jaynes - Cummings Hamiltonian

coupling strength $g = \frac{e}{\hbar}\frac{C_g}{C_\Sigma} V_0$

note, the term $a^\dagger\sigma_- + a\sigma_+$ only couples the term $|g, n\rangle$ and $|e, n-1\rangle$ i.e terms with n excitations

$$|+, n\rangle = \cos\theta_{JC}\, |e, n-1\rangle + \sin\theta_{JC}\, |g, n\rangle$$
$$|-, n\rangle = \sin\theta_{JC}\, |e, n-1\rangle - \sin\theta_{JC}\, |g, n\rangle$$

where θ_{JC} is the mixing angle

$$\tan(2\theta_{JC}) = \frac{2g\sqrt{n}}{\Delta} \qquad \Delta = \Omega - \omega_r$$

Eigenenergies

$$E_{\pm,n} = n\hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4g^2 n + \Delta^2}$$

dispersive regime

$$\Delta \gg g$$

$$|-,0\rangle \simeq \frac{g}{\Delta}|e,0\rangle - |g,1\rangle$$
$$|+,0\rangle \simeq |e,0\rangle + \frac{g}{\Delta}|g,1\rangle$$

making the unitary transformation

$$U = \exp\left[\frac{g}{\Delta}\left(a\sigma_+ - a^\dagger\sigma_-\right)\right]$$

second order in g/Δ

stark shift

$$UHU^\dagger \approx \hbar\underbrace{\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)}_{\text{qubit dependent resonator frequency}} a^\dagger a + \frac{\hbar}{2}\left(\Omega + \frac{g^2}{\Delta}\right)\sigma_z$$